



Multi-level SEM – part 1

SEM for between-subjects and mixed designs



Intro

Today's goal:

Teach how to do multi-level CFA and SEM in R and Mplus

Outline:

- Intro to Mplus (including common method bias!)
- Why multi-level?
- Multi-level CFA and SEM in R
- Multi-level CFA and SEM in Mplus



Intro to Mplus

Confirmatory Factor Analysis



Specifying a CFA

DATA: FILE IS twq.datm;

VARIABLE:

names are s1 s2 s3 s4 s5 s6 s7 q1 q2 q3 q4 q5 q6
c1 c2 c3 c4 c5 u1 u2 u3 u4 u5 cgraph citem cfriend;

usevariables are s1-s7 q1-q6 c1-c4 u2 u4 u5;

categorical are all;

MODEL:

satisf by s1-s7;
quality by q1-q6;
control by c1-c4;
underst by u2 u4 u5;

OUTPUT:

standardized;
modindices(3.84);



Run the model

Running the model:

- Save file as model.inp
- Run the model, this will create and open model.out
- Inspect model output file (for UVI results, look under STDYX, STDY, or STD)
- Iterate (not needed here)

Tip: When trimming, remove items from usevariables as well!



Method bias

To test common method bias, change the following:

MODEL:

```
satisf by s1-s7*;  
quality by q1-q6*;  
control by c1-c4*;  
underst by u2* u4 u5;
```

```
method by s1-u5* (a);  
satisf-method@1;  
method with satisf-underst@0;
```



Interpretation

We force UVI in the main model

Using stars to free up the first loading and satisfy-method@1 to set the factor variances to 1

We create a “method” factor with all items

They all load equally on the factor (a)

We make the factor orthogonal to the other factors
(method with satsif-underst@0)

The bias is significant, but not strong

It doesn't affect the factor model much



Specifying an SEM

VARIABLE:

```
names are s1 s2 s3 s4 s5 s6 s7 q1 q2 q3 q4 q5 q6  
c1 c2 c3 c4 c5 u1 u2 u3 u4 u5 cgraph citem cfriend;
```

```
usevariables are s1-s7 q1-q6 c1-c4 u2 u4 u5 cgraph citem  
cfriend;
```

```
categorical are s1-u5;
```

MODEL:

```
satisf by s1-s7;  
quality by q1-q6;  
control by c1-c4;  
underst by u2 u4 u5;
```

```
satisf on quality control;  
quality on control;  
control on underst;  
underst on cgraph citem cfriend (p1-p3);
```



Tips

Different outputs:

- Regular results: not standardized
- STDYX standardization: standardizes factors, and any single-time variables that are either x or y
- STDY standardization: same, but does not standardize single-item variables that are x (experimental conditions!)
- STD standardization: standardizes only the factors

Here, we use STDY

Or STD, same thing!



Tips

You can test omnibus effects like this:

MODEL TEST:

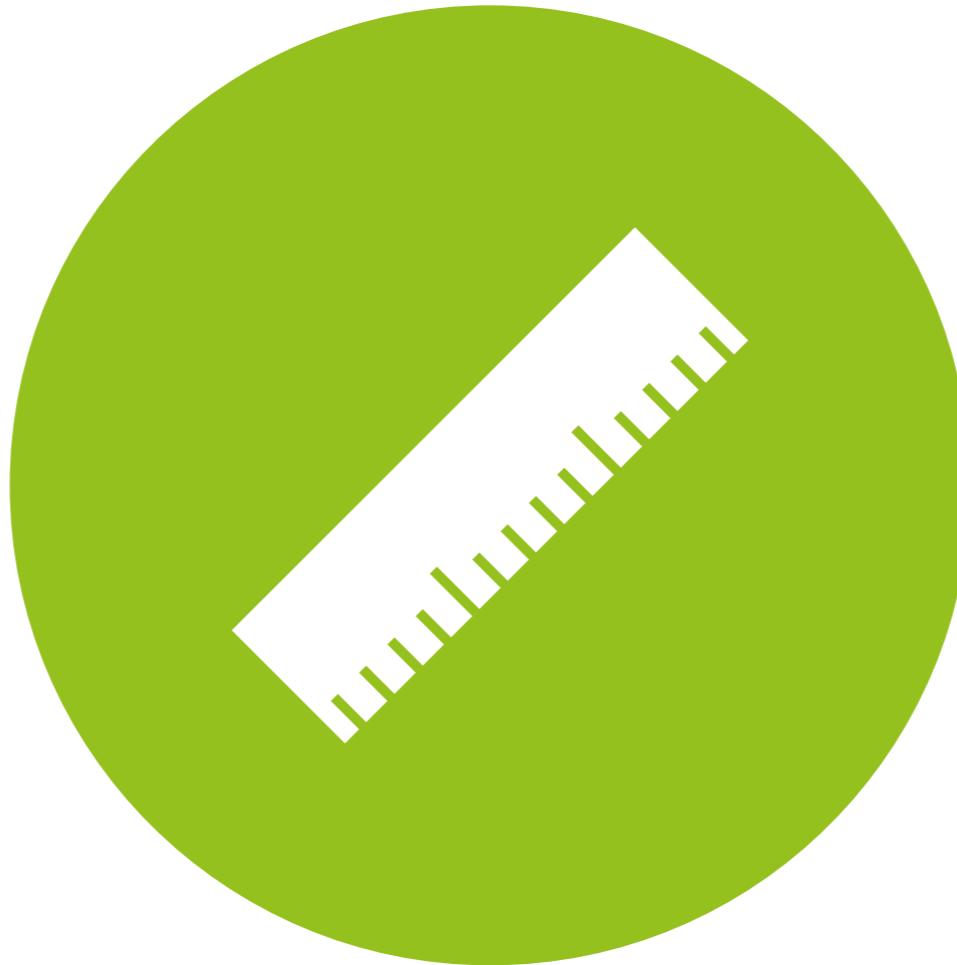
```
p2=0;  
p3=0;
```

Shows up under Wald Test of Parameter Constraints

You can create your own interaction effects:

DEFINE:

```
cig = citem*cgraph;  
cfg = cfriend*cgraph;
```



Why multi-level?

Dealing with within-subjects and mixed designs



Why multi-level?

Repeated measurements

e.g. participants make 30 decisions

(Partially) within-subjects design

e.g. participants are randomly assigned to 1 of 3 games,
and tested once with sound on and once with sound off

Grouped data

e.g. participants perform tasks in groups of 5

A combination of the above

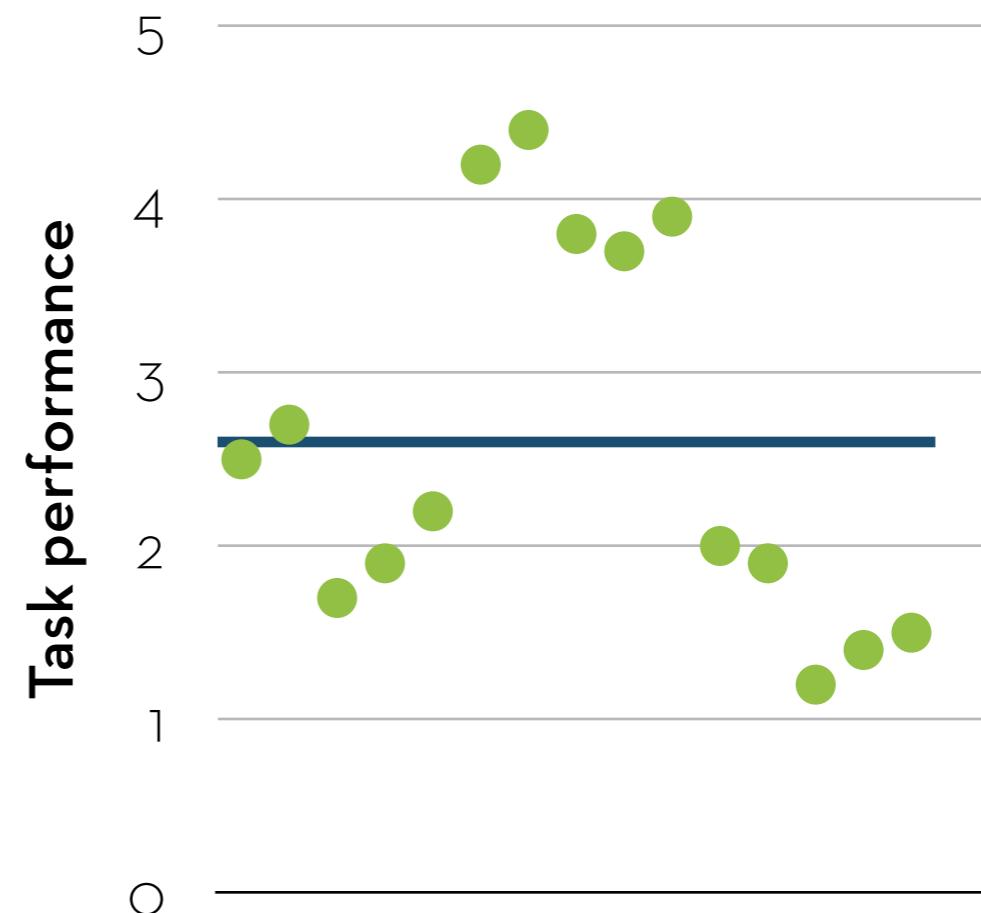


Correlated errors

Consequence: errors are correlated

There will be a user-bias
(and maybe an task-bias)

Golden rule: data-points should be **independent**



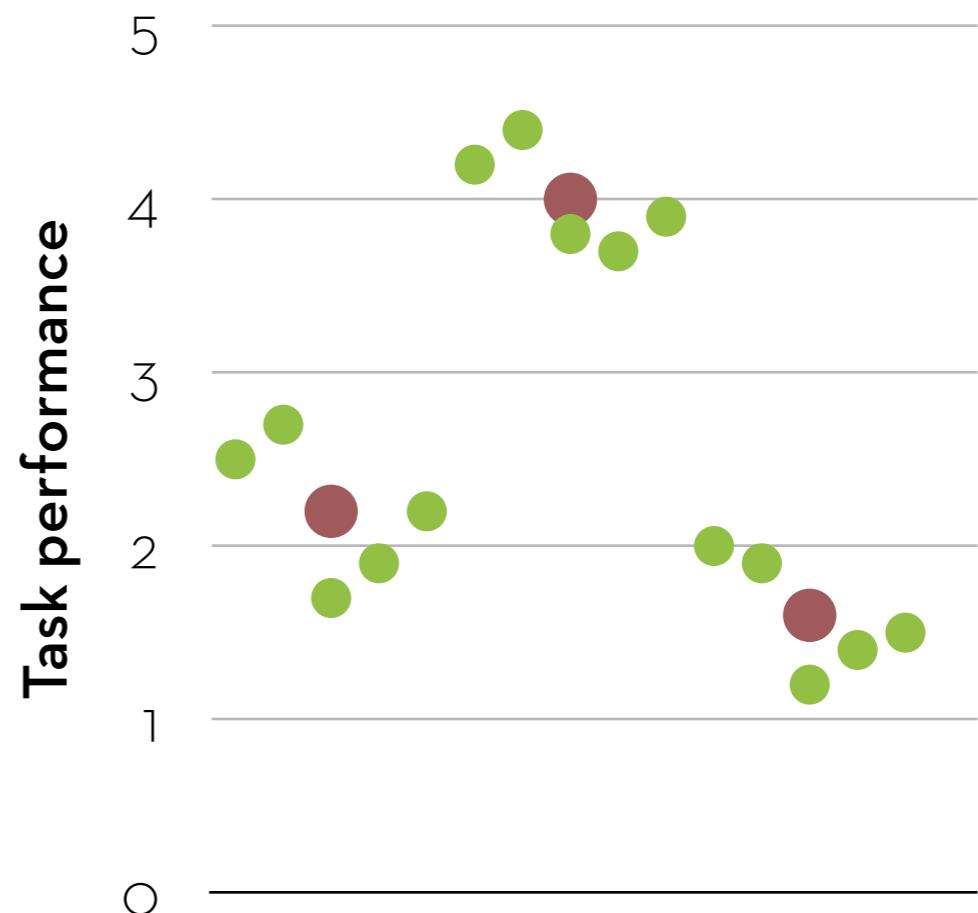


OK solution

Take the average of the repeated measurements

Reduces the number of observations

It becomes impossible to make inferences about individual tasks/users/etc.





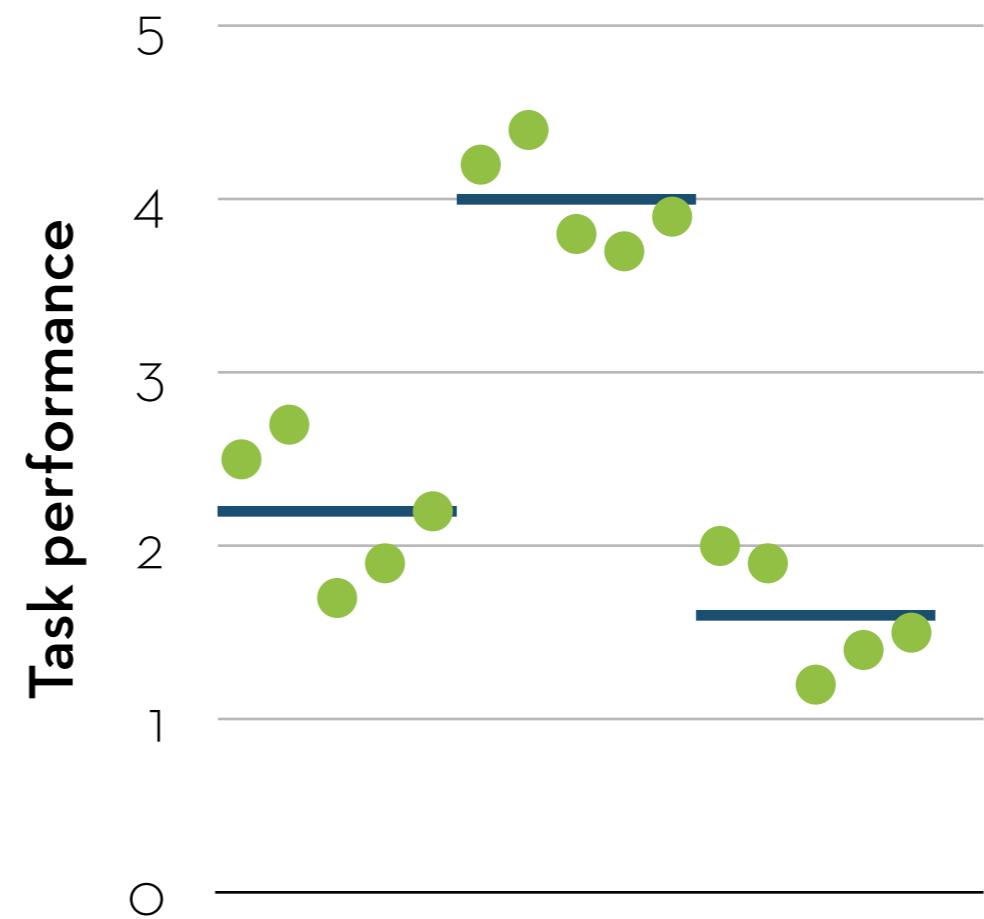
Good solution

In regression:

- define a random intercept for each user
- impose an error covariance structure
- see “multilevel” of M&E I

In SEM:

- use lavaan.survey (limited)
- use Mplus (better)





Multi-level in R

CFA and SEM



Multi-level in R

Dataset:

Participants rate a number of movies

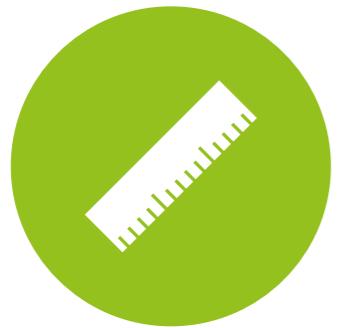
Participants are randomly assigned to a certain recommendation list length

5, 10, 15, 20, or 15 items

Participants get to see three recommendation lists

low, mid, and high diversification

In random order



Multi-level in R

Per participant:

length: length of recommendation list (5, 10, 15, 20, or 25)

pref1–4: strength of preference (4 items, 7pt scales)

exp1–4: movie expertise



Multi-level in R

Per list:

divmid, divhigh: dummies for diversification of recommendation list (baseline: low)

chodiff: choice difficulty (single item, 7pt scale)

to: tradeoff difficulty (single item, 7pt scale)

var1–4: perceived recommendation diversity (7pt scales)

acc1–6: perceived recommendation attractiveness



Specifying a CFA

Specify the model:

```
model <- "acc =~ acc1+acc2+acc3+acc4+acc5+acc6  
var =~ var1+var2+var3+var4  
pref =~ pref1+pref2+pref3+pref4  
exp =~ exp1+exp2+exp3+exp4"
```

Run the model:

```
fit <- cfa(model, ordered=names(divR), data=divR, std.lv=T)
```

Get the output:

```
summary(fit, rsquare=T, fit.measures=T)
```

Modification indices:

```
mods <- modindices(fit, power=T)  
mods <- mods[grep("\\*", mods$decision), ]
```



Selected output

Estimator	DWLS	Robust
Minimum Function Test Statistic	487.089	498.526
Degrees of freedom	129	129
P-value (Chi-square)	0.000	0.000
Scaling correction factor		1.121
Shift parameter for simple second-order correction (Mplus variant)		64.179

User model versus baseline model:

Comparative Fit Index (CFI)	0.991	0.978
Tucker-Lewis Index (TLI)	0.990	0.974
Robust Comparative Fit Index (CFI)		NA
Robust Tucker-Lewis Index (TLI)		NA

Root Mean Square Error of Approximation:

RMSEA	0.098	0.099
90 Percent Confidence Interval	0.089	0.107
P-value RMSEA <= 0.05	0.000	0.000
Robust RMSEA		NA
90 Percent Confidence Interval		NA NA



Selected output

R-Square:

	Estimate
acc1	0.834
acc2	0.677
acc3	0.945
acc4	0.696
acc5	0.891
acc6	0.593
var1	0.837
var2	0.662
var3	0.557
var4	0.635
pref1	0.782
pref2	0.864
pref3	0.266
pref4	0.047
exp1	0.688
exp2	0.905
exp3	0.704
exp4	0.688



Selected output

	lhs	op	rhs	mi	mi.scaled	epc	sepc.all	delta	ncp	power	decision
192	acc	=~	var1	24.165	21.549	0.157	0.157	0.1	9.800	0.879	*epc:m*
194	acc	=~	var3	53.095	47.346	0.232	0.232	0.1	9.878	0.882	*epc:m*
199	acc	=~	pref4	18.791	16.756	0.101	0.101	0.1	18.498	0.990	*epc:m*
205	var	=~	acc2	26.506	23.636	0.171	0.171	0.1	9.093	0.854	*epc:m*
209	var	=~	acc6	14.523	12.950	0.150	0.150	0.1	6.431	0.718	**(m)**
210	var	=~	pref1	3.940	3.513	0.082	0.082	0.1	5.821	0.675	**(m)**
213	var	=~	pref4	13.765	12.274	0.104	0.104	0.1	12.686	0.945	*epc:m*
221	pref	=~	acc4	9.688	8.639	-0.122	-0.122	0.1	6.463	0.720	**(m)**
229	pref	=~	exp2	4.337	3.867	-0.108	-0.108	0.1	3.747	0.490	**(m)**
231	pref	=~	exp4	5.719	5.100	-0.108	-0.108	0.1	4.868	0.597	**(m)**
240	exp	=~	var3	11.609	10.352	0.112	0.112	0.1	9.269	0.861	*epc:m*
242	exp	=~	pref1	5.587	4.982	0.132	0.132	0.1	3.193	0.431	**(m)**
243	exp	=~	pref2	6.881	6.136	0.159	0.159	0.1	2.730	0.379	**(m)**
244	exp	=~	pref3	11.836	10.555	0.128	0.128	0.1	7.244	0.768	*epc:m*
245	exp	=~	pref4	29.025	25.882	0.211	0.211	0.1	6.528	0.724	**(m)**
248	acc1	~~	acc4	16.956	15.120	0.142	0.142	0.1	8.447	0.828	*epc:m*
250	acc1	~~	acc6	15.765	14.058	-0.168	-0.168	0.1	5.558	0.655	**(m)**
253	acc1	~~	var3	13.565	12.096	0.188	0.188	0.1	3.844	0.500	**(m)**
264	acc2	~~	acc4	8.077	7.203	-0.121	-0.121	0.1	5.476	0.648	**(m)**
266	acc2	~~	acc6	19.095	17.027	0.219	0.219	0.1	3.969	0.513	**(m)**
268	acc2	~~	var2	7.373	6.575	-0.123	-0.123	0.1	4.891	0.599	**(m)**
269	acc2	~~	var3	29.199	26.037	-0.268	-0.268	0.1	4.067	0.523	**(m)**
270	acc2	~~	var4	6.273	5.593	0.095	0.095	0.1	6.906	0.748	**(m)**



Multi-level CFA

Install lavaan.survey

Set up the random intercept:

```
divR.design <- svydesign(ids= ~userId, data=divR)
```

Re-run the model with the survey design:

```
fit.multi <- lavaan.survey(fit, divR.design)
```

Get the output:

```
summary(fit.multi, rsquare=T, fit.measures=T)
```

Modification indices:

```
mods <- modindices(fit.multi, power=T)
mods <- mods[grep("\\*", mods$decision), ]
```



Selected output

Estimator	ML	Robust
Minimum Function Test Statistic	501.651	263.380
Degrees of freedom	129	129
P-value (Chi-square)	0.000	0.000
Scaling correction factor for the Satorra–Bentler correction		1.905

User model versus baseline model:

Comparative Fit Index (CFI)	0.895	0.903
Tucker–Lewis Index (TLI)	0.875	0.885
Robust Comparative Fit Index (CFI)		0.923
Robust Tucker–Lewis Index (TLI)		0.909

Root Mean Square Error of Approximation:

RMSEA	0.100	0.060
90 Percent Confidence Interval	0.091	0.109
P-value RMSEA <= 0.05	0.000	0.016
Robust RMSEA		0.083
90 Percent Confidence Interval		0.068 0.097



Selected output

R-Square:

	Estimate
acc1	0.825
acc2	0.667
acc3	0.918
acc4	0.494
acc5	0.857
acc6	0.333
var1	0.739
var2	0.614
var3	0.562
var4	0.586
pref1	0.746
pref2	0.742
pref3	0.257
pref4	0.046
exp1	0.654
exp2	0.867
exp3	0.653
exp4	0.570



Selected output

	lhs	op	rhs	mi	mi.scaled	epc	sepc.all	delta	ncp	power	decision
69	acc	=~	var1	12.639	6.636	0.229	0.158	0.1	2.407	0.342	**(m)**
71	acc	=~	var3	24.447	12.835	0.345	0.240	0.1	2.059	0.300	**(m)**
82	var	=~	acc2	9.295	4.880	0.202	0.126	0.1	2.275	0.326	**(m)**
84	var	=~	acc4	16.264	8.539	-0.331	-0.202	0.1	1.488	0.230	**(m)**
86	var	=~	acc6	11.341	5.955	0.295	0.191	0.1	1.301	0.207	**(m)**
93	var	=~	exp3	4.275	2.245	0.131	0.083	0.1	2.481	0.350	**(m)**
106	pref	=~	exp2	11.565	6.072	-0.238	-0.139	0.1	2.034	0.297	**(m)**
108	pref	=~	exp4	5.978	3.138	-0.178	-0.124	0.1	1.882	0.279	**(m)**
112	exp	=~	acc4	3.858	2.026	-0.148	-0.090	0.1	1.764	0.264	**(m)**
122	exp	=~	pref4	5.501	2.888	0.216	0.161	0.1	1.174	0.192	**(m)**
125	acc1	~~	acc4	6.200	3.255	0.138	0.050	0.1	3.255	0.438	**(m)**
126	acc1	~~	acc5	5.422	2.847	0.098	0.035	0.1	5.654	0.662	**(m)**
138	acc1	~~	exp3	10.174	5.342	0.146	0.055	0.1	4.801	0.591	**(m)**
139	acc1	~~	exp4	6.048	3.175	0.111	0.046	0.1	4.874	0.598	**(m)**
140	acc2	~~	acc3	11.760	6.174	-0.146	-0.053	0.1	5.545	0.654	**(m)**
143	acc2	~~	acc6	5.294	2.779	0.166	0.067	0.1	1.932	0.285	**(m)**
146	acc2	~~	var3	5.870	3.082	-0.141	-0.061	0.1	2.961	0.406	**(m)**
150	acc2	~~	pref3	4.830	2.536	0.128	0.068	0.1	2.938	0.403	**(m)**
152	acc2	~~	exp1	4.999	2.624	0.118	0.050	0.1	3.584	0.473	**(m)**
172	acc4	~~	acc6	110.831	58.189	-0.935	-0.370	0.1	1.267	0.203	**(m)**
174	acc4	~~	var2	6.941	3.644	0.178	0.077	0.1	2.200	0.317	**(m)**
184	acc4	~~	exp4	7.039	3.695	0.185	0.078	0.1	2.068	0.301	**(m)**
211	var1	~~	var3	13.731	7.209	0.291	0.139	0.1	1.623	0.247	**(m)**



Trimming

Remove pref4, acc4, and acc6:

```
model <- "acc =~ acc1+acc2+acc3+acc5  
var =~ var1+var2+var3+var4  
pref =~ pref1+pref2+pref3  
exp =~ exp1+exp2+exp3+exp4"
```

Run the model, then rerun it with the survey design:

```
fit <- cfa(model, ordered=names(divR), data=divR, std.lv=T)  
fit.multi <- lavaan.survey(fit, divR.design)
```

Get the output and modification indices:

```
summary(fit.multi, rsquare=T, fit.measures=T)  
mods <- modindices(fit.multi, power=T)  
mods <- mods[grep("\\*", mods$decision), ]
```



Specifying an SEM

Specify the model:

```
model <- "acc =~ acc1+acc2+acc3+acc5  
var =~ var1+var2+var3+var4  
pref =~ pref1+pref2+pref3  
exp =~ exp1+exp2+exp3+exp4  
  
chodiff ~ to+acc+var+length  
to ~ divmid+divhigh+pref  
acc ~ var+divmid+divhigh+exp  
var ~ divmid+divhigh"
```

Run the model:

```
fit <- sem(model, ordered=names(divR[5:24]), data=divR, std.lv=T)
```

Get the output:

```
summary(fit, rsquare=T, fit.measures=T)
```



Selected output

Estimator	DWLS	Robust
Minimum Function Test Statistic	274.599	265.706
Degrees of freedom	158	158
P-value (Chi-square)	0.000	0.000
Scaling correction factor		1.500
Shift parameter for simple second-order correction (Mplus variant)		82.594

User model versus baseline model:

Comparative Fit Index (CFI)	0.997	0.993
Tucker-Lewis Index (TLI)	0.996	0.992
Robust Comparative Fit Index (CFI)		NA
Robust Tucker-Lewis Index (TLI)		NA

Root Mean Square Error of Approximation:

RMSEA	0.050	0.048
90 Percent Confidence Interval	0.040	0.060
P-value RMSEA <= 0.05	0.458	0.586
Robust RMSEA		NA
90 Percent Confidence Interval		NA NA



Selected output

Regressions:

		Estimate	Std.Err	z-value	P(> z)
chodiff ~					
to		0.352	0.045	7.827	0.000
acc		-0.233	0.045	-5.188	0.000
var		-0.240	0.054	-4.453	0.000
length		0.017	0.009	1.979	0.048
to ~					
divmid		-0.049	0.153	-0.318	0.750
divhigh		-0.348	0.149	-2.327	0.020
pref		-0.147	0.059	-2.511	0.012
acc ~					
var		0.361	0.052	6.985	0.000
divmid		0.244	0.160	1.524	0.127
divhigh		0.179	0.159	1.127	0.260
exp		0.242	0.062	3.897	0.000
var ~					
divmid		0.320	0.159	2.018	0.044
divhigh		0.515	0.152	3.389	0.001

Covariances:

		Estimate	Std.Err	z-value	P(> z)
pref ~~					
exp		0.435	0.045	9.632	0.000

note: to and chodiff are not standardized!



Multi-level SEM

Re-run the model with the survey design:

```
fit.multi <- lavaan.survey(fit, divR.design)
```

Get the output:

```
summary(fit.multi, rsquare=T, fit.measures=T)
```



Selected output

Estimator	ML	Robust
Minimum Function Test Statistic	302.464	187.340
Degrees of freedom	158	158
P-value (Chi-square)	0.000	0.055
Scaling correction factor for the Satorra–Bentler correction		1.615

User model versus baseline model:

Comparative Fit Index (CFI)	0.953	0.980
Tucker–Lewis Index (TLI)	0.945	0.977
Robust Comparative Fit Index (CFI)		0.984
Robust Tucker–Lewis Index (TLI)		0.981

Root Mean Square Error of Approximation:

RMSEA	0.056	0.025
90 Percent Confidence Interval	0.046	0.066
P-value RMSEA <= 0.05		1.000
Robust RMSEA		0.032
90 Percent Confidence Interval		NA 0.049



Selected output

Regressions:

		Estimate	Std.Err	z-value	P(> z)
chodiff ~					
to		0.351	0.079	4.413	0.000
acc		-0.361	0.127	-2.851	0.004
var		-0.381	0.129	-2.954	0.003
length		0.019	0.016	1.181	0.237
to ~					
divmid		-0.031	0.164	-0.189	0.850
divhigh		-0.412	0.166	-2.486	0.013
pref		-0.144	0.112	-1.285	0.199
acc ~					
var		0.389	0.093	4.164	0.000
divmid		0.248	0.121	2.045	0.041
divhigh		0.186	0.137	1.355	0.175
exp		0.270	0.088	3.074	0.002
var ~					
divmid		0.364	0.126	2.900	0.004
divhigh		0.530	0.150	3.522	0.000

Covariances:

		Estimate	Std.Err	z-value	P(> z)
pref ~~					
exp		0.439	0.108	4.073	0.000



Multi-level in R

Downsides:

- Having to run the model twice is awkward
- Items are treated as ratio variables rather than ordinal
(this sucks for 5-point scales)
- Advanced tricks with random slopes etc. not possible
(more on this next week)

Solution:

Use Mplus, which has native support for multi-level models



Multi-level in Mplus

CFA and SEM



Multi-level in Mplus

Data preparation:

- Remove heading row from divR.csv file, save as divM.csv
- Make a new file in MPlus with the dataset and model definition
- Save file as model.inp
- Run the model, this will create and open model.out
- Inspect model output file



Specifying a CFA

DATA: FILE IS divM.csv;

VARIABLE:

names are userId divmid divhigh length chodiff to
var1 var2 var3 var4 acc1 acc2 acc3 acc4 acc5 acc6
pref1 pref2 pref3 pref4 exp1 exp2 exp3 exp4;

usevariables are var1 var2 var3 var4 acc1 acc2 acc3 acc4 acc5 acc6
pref1 pref2 pref3 pref4 exp1 exp2 exp3 exp4;

categorical are all;

cluster = userId;

ANALYSIS:

type is complex;

MODEL:

acc by acc1–acc6;
var by var1–var4;
pref by pref1–pref4;
exp by exp1–exp4;

OUTPUT:

standardized;
modindices(all);



Selected output

Chi-Square Test of Model Fit

Value	293.063*
Degrees of Freedom	129
P-Value	0.0000

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.066
90 Percent C.I.	0.056 0.076
Probability RMSEA <= .05	0.005

CFI/TLI

CFI	0.979
TLI	0.976



Selected output

R-SQUARE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	Residual Variance
VAR1	0.855	0.038	22.376	0.000	0.145
VAR2	0.665	0.044	15.036	0.000	0.335
VAR3	0.556	0.044	12.560	0.000	0.444
VAR4	0.627	0.043	14.624	0.000	0.373
ACC1	0.809	0.025	32.693	0.000	0.191
ACC2	0.684	0.037	18.553	0.000	0.316
ACC3	0.923	0.011	82.658	0.000	0.077
ACC4	0.706	0.049	14.397	0.000	0.294
ACC5	0.917	0.020	45.548	0.000	0.083
ACC6	0.580	0.051	11.304	0.000	0.420
PREF1	0.782	0.091	8.632	0.000	0.218
PREF2	0.862	0.086	10.028	0.000	0.138
PREF3	0.269	0.081	3.318	0.001	0.731
PREF4	0.049	0.046	1.067	0.286	0.951
EXP1	0.680	0.065	10.517	0.000	0.320
EXP2	0.901	0.058	15.567	0.000	0.099
EXP3	0.710	0.063	11.331	0.000	0.290
EXP4	0.695	0.077	9.027	0.000	0.305



Selected output

MODEL MODIFICATION INDICES

WITH Statements

VAR1	WITH ACC	14.524	0.128	0.142	0.372
VAR1	WITH VAR	14.566	-0.326	-0.353	-0.926
VAR3	WITH ACC	27.635	0.178	0.198	0.298
VAR3	WITH VAR	27.964	-0.458	-0.496	-0.744
VAR3	WITH VAR2	12.708	0.193	0.193	0.501
ACC2	WITH ACC	18.492	-0.330	-0.367	-0.653
ACC2	WITH VAR	15.597	0.140	0.151	0.269
ACC2	WITH VAR3	14.072	-0.273	-0.273	-0.729
ACC3	WITH ACC2	11.073	-0.115	-0.115	-0.740
ACC4	WITH ACC	20.035	0.370	0.411	0.760
ACC4	WITH VAR	11.487	-0.133	-0.144	-0.265
ACC6	WITH ACC4	45.503	-0.300	-0.300	-0.853
PREF4	WITH PREF3	15.281	0.430	0.430	0.516



SEM (trimmed)

DATA: FILE IS divM.csv;

VARIABLE:

names are userId divmid divhigh length chodiff to
var1 var2 var3 var4 acc1 acc2 acc3 acc4 acc5 acc6
pref1 pref2 pref3 pref4 exp1 exp2 exp3 exp4;

usevariables are **divmid divhigh length chodiff to**
var1 var2 var3 var4 acc1 acc2 acc3 acc5
pref1 pref2 pref3 exp1 exp2 exp3 exp4;

categorical are var1-exp4;
cluster = userId;

ANALYSIS:

type is complex;

MODEL:

acc by acc1-**acc5**;
var by var1-var4;
pref by pref1-**pref3**;
exp by exp1-exp4;

chodiff on to acc var length;
to on divmid divhigh pref;
acc on var divmid divhigh exp;
var on divmid divhigh;

OUTPUT: standardized;



Selected output

Chi-Square Test of Model Fit

Value	199.864*
Degrees of Freedom	158
P-Value	0.0135

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used

for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV,
and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.030
90 Percent C.I.	0.015 0.042
Probability RMSEA <= .05	0.998

CFI/TLI

CFI	0.995
TLI	0.994



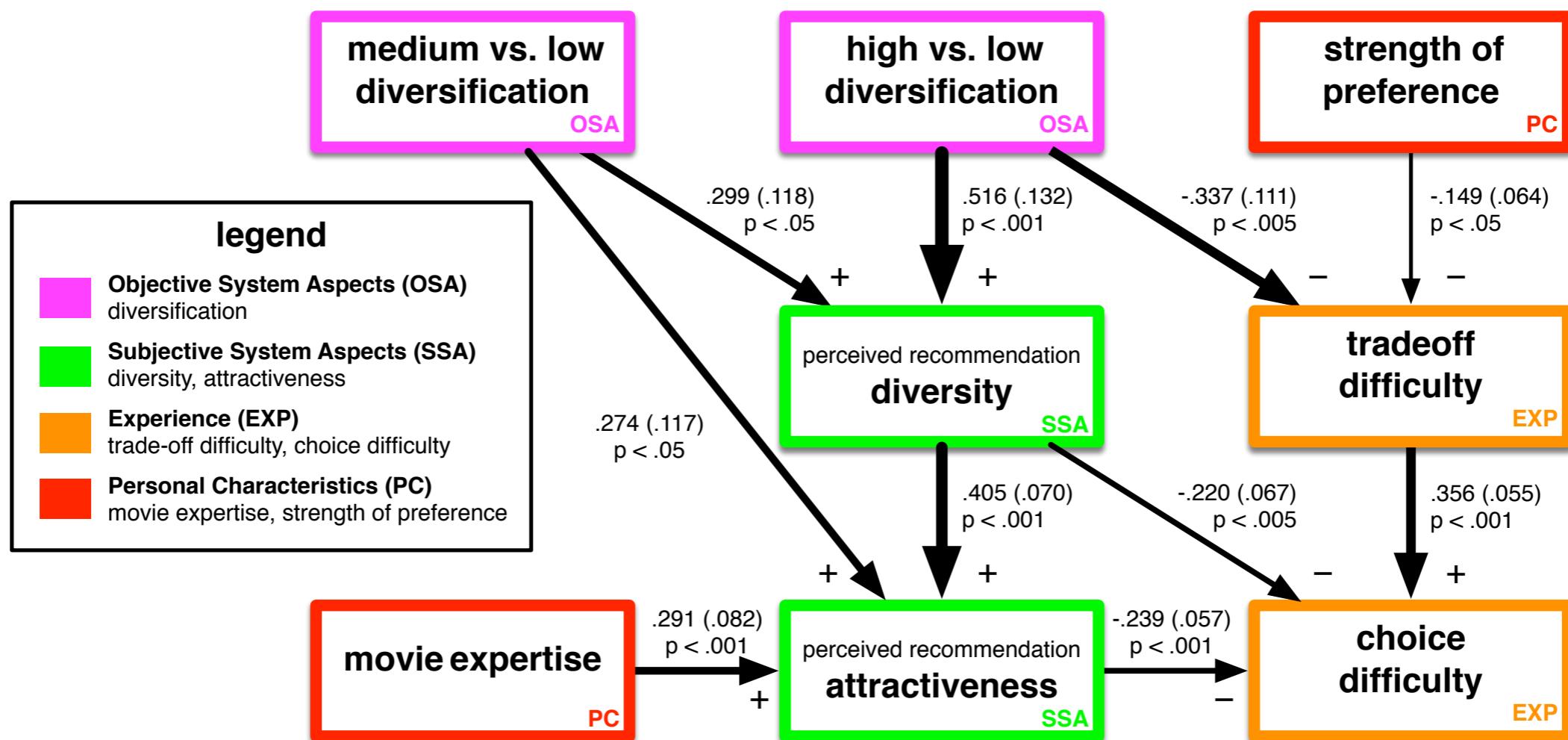
Selected output

ACC	ON			
VAR		0.336	0.052	6.436
EXP		0.218	0.069	3.159
ACC	ON			
DIVMID		0.217	0.106	2.056
DIVHIGH		0.157	0.116	1.349
VAR	ON			
DIVMID		0.311	0.115	2.700
DIVHIGH		0.504	0.126	3.991
CHODIFF	ON			
ACC		-0.274	0.069	-3.944
VAR		-0.206	0.063	-3.255
T0	ON			
PREF		-0.148	0.069	-2.139
T0	ON			
T0		0.365	0.069	5.254
LENGTH		0.014	0.011	1.322
T0	ON			
DIVMID		-0.047	0.116	-0.408
DIVHIGH		-0.314	0.110	-2.844
				0.683
				0.004

note: using STDY standardization... why?



Final model





Total effects? Easy!

```
MODEL INDIRECT:  
acc IND divmid;  
acc IND divhigh;  
chodiff IND divmid;  
chodiff IND divhigh;
```

**“It is the mark of a truly intelligent person
to be moved by statistics.”**

T

H

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S

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George Bernard Shaw